1. For each function below, tell me whether or not it is one-to-one, and whether or not it is onto:

(a) \( f : \mathbb{Z} \to \mathbb{Z} \) by \( f(n) = n + 5 \)
(b) \( f : \mathbb{Z} \to \mathbb{Z} \) by \( f(n) = 2n + 5 \)
(c) \( f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \) by \( f((m, n)) = mn \)
(d) \( f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \) by \( f((m, n)) = 2mn \)

2. Consider a function \( f : A \to B \) with formula \( f(x) = x^2 \).

(a) Define sets \( A \) and \( B \) so that \( f \) is one-to-one and onto.
(b) Define sets \( A \) and \( B \) so that \( f \) is one-to-one, but not onto.
(c) Define sets \( A \) and \( B \) so that \( f \) is onto, but not one-to-one.
(d) Define sets \( A \) and \( B \) so that \( f \) is neither one-to-one nor onto.

3. Define a function \( f : \{-2, -1, 0, 1, 2, 3, \ldots, 10\} \to \mathbb{R} \) by \( f(x) = x^2 \).

(a) What is \( f(\{-1, 1\}) \)?
(b) What is \( f(-1, 1) \)?
(c) What is \( f((-1, 1)) \)?
(d) What is \( f(\emptyset) \)?
(e) What is \( f^{-1}(\{-1, 1\}) \)?
(f) What is \( f^{-1}(\{9\}) \)?
(g) What is \( f^{-1}(9) \)?
(h) What is \( f^{-1}(\{2\}) \)?
(i) What is \( f^{-1}(2) \)?
(j) What is \( f(f^{-1}(\{4\})) \)?
(k) What is \( f(f^{-1}(\{3\})) \)?
(l) What is \( f^{-1}(f(2)) \)?
(m) What is \( f^{-1}(f(\{2\})) \)?

4. Define a function \( f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \) by \( f((a, b)) = a + b \).

(a) What is \( f^{-1}(2) \)?
(b) What is \( f^{-1}(\{2\}) \)?