1. The exam begins at 9:00 a.m. and ends at noon.

2. Calculators, books, notes, and communication with others are prohibited.

3. You must provide full justification for your answers.

4. For each problem, write your answer on a separate piece of paper and include your name and the number of the question at the top of the paper.

5. Omit one of the 15 questions by clearly marking through it with an X.

1. Compute each limit or show that it does not exist:
   
   (a) \( \lim_{x \to 0^+} \frac{|x|}{x} \)
   
   (b) \( \lim_{x \to 0^-} \frac{|x|}{x} \)
   
   (c) \( \lim_{x \to 0} \frac{|x|}{x} \)
   
   (d) \( \lim_{x \to 0^-} \frac{\sin(5x)}{x} \)
   
   (e) \( \lim_{x \to 0} \frac{\sin(5x)}{x} \)

2. Suppose that \( D(a) \) represents the average stopping distance (in feet) of a Ford Fiesta with tires that are \( a \) years old. Is the function \( D' \)
   
   • always positive or zero;
   
   • always negative or zero;
   
   • sometimes positive and sometimes negative?

   Explain.

3. For the function \( f(x) = xe^{-x} \),
   
   (a) Find the interval(s) on which the graph of \( f \) is increasing;
   
   (b) Find the interval(s) on which the graph of \( f \) is decreasing;
   
   (c) Find the interval(s) on which the graph of \( f \) is concave up;
   
   (d) Find the interval(s) on which the graph of \( f \) is concave down;

4. Evaluate each integral:
   
   (a) \( \int_0^\pi t^2 \cos(t) \, dt \)
   
   (b) \( \int_0^\pi t \cos(t^2) \, dt \)

5. Consider the infinite series: \( \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2} \). Estimate the sum of this series, accurate to within .05. Explain how you know you have achieved the desired accuracy.

6. Explain what a Taylor series is and why we might be interested in such a thing. Give a few examples.

7. Let \( f(x, y) = x^2 + y^2 \). Compute \( \int_R f \, dA \), where \( R \) is the region shown here:
8. Let \( g(x, y) = x^2 + 2y \).
   (a) Compute the gradient of \( g \) at the point \((1, 0)\).
   (b) Explain how your answer to (a) is related to the contour plot for \( g \) and what it says about the graph of \( g \).

9. Consider the cube with corners at \((0, 0, 0), (1, 0, 0), (0, 1, 0), (1, 1, 0), (0, 0, 1), (0, 1, 1)\), and \((1, 1, 1)\). Let \( S \) denote the surface of that cube, oriented outward. Now consider the vector field \( \vec{F}(x, y, z) = (5x, 4y, -4z) = 5x\vec{i} + 4y\vec{j} - 4z\vec{k} \).
   Using one of your favorite big theorems, it is extremely easy to compute the surface integral of \( \vec{F} \) over \( S \). Do it.

10. (a) Describe the three row operations that are used to row-reduce matrices.
    (b) For a square matrix, describe how each elementary row operation affects the determinant.

11. Consider the vectors \( \vec{u} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \), \( \vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \), and \( \vec{w} = \begin{pmatrix} 1 \\ 5 \\ 8 \end{pmatrix} \).
    (a) Show that these three vectors form a basis for \( \mathbb{R}^3 \).
    (b) Use the Gram-Schmidt process to convert this basis to an orthogonal basis.

12. Suppose \( A \) is a matrix with 6 rows and 9 columns.
    (a) Explain why \( A \) cannot be invertible.
    (b) What are the possible dimensions of the null space of \( A \)? For each possibility, what will be the dimension of the column space of \( A \)?

13. Prove by induction that 3 is a divisor of \( n^3 + 2n \) for every positive integer \( n \).

14. How many positive integers between 1 and 1000 (inclusive) are:
    (a) divisible by 7?
    (b) divisible by 11?
    (c) divisible by both 7 and 11?
    (d) divisible by either 7 or 11?
    (e) divisible by exactly one of 7 or 11?
    (f) divisible by neither 7 nor 11?

15. Define a relation \( \sim \) on the integers by \( a \sim b \) if and only if \( a - b \) is divisible by 7. Is \( \sim \) an equivalence relation? Prove or disprove.